A local planner for closed-loop robot

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***Abstract*— Global motion planners have been proposed for closed-loop robot based on the same paradigm than has been proposed for serial chains. First a sparse representation of the configuration space of the robot is constructed as a set of nodes. This is somewhat more complicated than for serial chain as the closure equations of the mechanism should be satisfied. Then a motion planning query consists simply in connecting the start and goal points through an appropriate set of nodes (usually minimizing the length of the trajectory). But such motion planner should be complemented by a local motion planner that addresses the following issues:**

1) **ensure that two successive nodes belong to the same robot kinematic branch (otherwise connecting these nodes will require to disassemble the robot)**
2) **verify that all poses between nodes satisfy the robot constraints (if possible taking into account the uncertainties in the robot modeling)**
3) **eventually try to shorten the trajectory length**

We present such a local motion planner that addresses all three issues and illustrates its use on a Gough parallel robot.

I. Introduction

A. Global motion planner

Motion planning is a classical problem in robotics and has been largely addressed for serial chains [2], [9], [10]. Among the most successful method we may mention the *roadmap* approach: a representative, but limited, set of reachable poses, called the sampling tree, is pre-computed and a planning query consists in connecting the reachable poses so that, for example, the length of the trajectory is minimal. An interest of this approach is that the construction of the sampling tree may be done once off-line, while finding a trajectory relies on determining a shortest path within a graph, a task for which there are efficient and fast algorithms. For serial chains the sampling is performed usually in the joint space (a point in this space leads to a unique configuration of the robot). Unfortunately adapting this approach to closed-loop chains is not easy as the joint variables should satisfy the *closure equations* and cannot be arbitrary chosen.

B. Closed-loop robots and kinematic constraints

The planning problem for serial chains is usually related to avoiding obstacles and self-collision, while for closed-chains *kinematic constraints* become preponderant.

A specific kinematic constraint for closed-loop structure is that their operational configuration space may have different components that are not connected. This may be illustrated simply on a four-bar mechanism (figure 1).

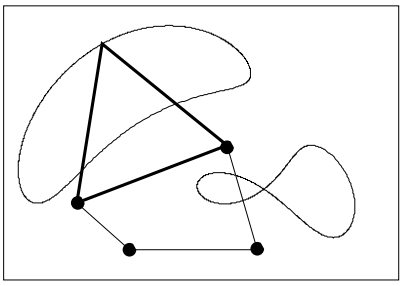

Fig. 1. A 4-bar mechanism with an operational space having two not connected components. Going from a pose in a component to another in the second component requires to disassemble the mechanism.

A consequence is that even if a sampling method is able to determine poses in the operational space that satisfy the closure equations, they may still belong to different *kinematic branches* and therefore cannot be connected by a trajectory.

Other kinematic constraints that must be satisfied are joint limits and collision avoidance between the robot's bodies. Another kinematic constraint is the absence of singularity on the trajectory. All these constraints are functions of the parameters that describe the geometry of the robot and must be satisfied although there are *uncertainties* on these parameters.

The complexity of the kinematic constraints and the uncertainties leads to complicated configuration space. In some cases this configuration space can be constructed either analytically [17] or numerically [12] but manipulating these representations for motion planning is a complicated task. The most successful motion planning method for closed-chains is called the *Probabilistic Roadmap* [4], [18], [19]. Here the sampling poses are chosen at random although general geometric algorithms are used to guide the sampling so that the poses satisfy the closure equations.

C. Local planner

In all cases these global motion planners must be complemented by a *local motion planner* that addresses the following issues:

1) ensure that two successive nodes in the trajectory belong to the same *kinematic branch* of the robot.
2) find a path that connect two successive nodes of the trajectory, such that any pose on the path are guaranteed to satisfy the kinematic constraints
3) eventually improve the trajectory in terms of some optimality criteria (e.g. its length) and provide a

solution that is "close" to the global optimum of the criteria (we will precise this closeness issue later on)

II. Basic ingredients

The local planner we have developed relies on the following assumptions:

- being given a pose of the end-effector we are able to state if the kinematic constraints are satisfied
- all the kinematic constraint may be written as $\mathcal{C}(\mathbf{X}, \mathbf{\Theta}) \leq 0$, where $\mathbf{X}$ is a n-dimensional vector constituted of the parameters that describes the end-effector pose and $\mathbf{\Theta}$ the joint variables (active and/or passive)
- the workspace of the robot is bounded

On the other hand we don't impose any particular form for $\mathcal{C}$ or any choice on the parameters $\mathbf{X}$.

The output of the planner will be a list of *way points* in the operational space, the start point S being the first element of the list while the goal point G will be the last element (S, G will be typically poses provided by the global planner). A *trajectory* between two way points $\mathbf{W_j}, \mathbf{W_{j+1}}$ will be defined by a set $\mathbf{F}$ of n continuous, analytical functions F_k such that any pose $\mathbf{X_t}$ on the trajectory may be written as $\mathbf{X_t} = \mathbf{F}(\mathbf{W_j}, \mathbf{W_{j+1}}, t)$, where t is the time and lie in the range [0,1] and such that $\mathbf{F}(\mathbf{W_j}, \mathbf{W_{j+1}}, 0) = \mathbf{W_j}$, $\mathbf{F}(\mathbf{W_j}, \mathbf{W_{j+1}}, 1) = \mathbf{W_{j+1}}$. At time t the k-th component of $\mathbf{X_t}$ will be obtained as $F_k(\mathbf{W_{j_k}}, \mathbf{W_{j+1_k}}, t)$. Any choice of F_k is possible as soon as it is possible to formulate analytically the optimality criteria for the trajectory (if any is used). For the sake of simplicity we will use here linear functions i.e. $\mathbf{X_t} = \mathbf{W_j} + t(\mathbf{W_{j+1}} - \mathbf{W_j})$.

The number of way points may not be defined in advance. If an optimality criteria is used, the final number of way points of the trajectory will be such that adding a new way point will not improve significantly the value of the optimality criteria. Note however that this local planner is only able to provide a limited number of way points (typically 3). Requiring more way points will mean that the global planner has performed poorly.

Any pose belonging to the trajectory between two way points will be guaranteed to satisfy the kinematic constraints. This is evidently a key issue that is addressed in the next section.

A. Checking a trajectory between two way points

We have already proposed a method to check if a given trajectory, defined by analytical function of the time, satisfies the kinematic constraints [11] and we will just outline its principle. We rely on *interval analysis* to determine if a kinematic constraint $\mathcal{C}_i$ is satisfied or violated over a given time range. Being given an analytical function $f(x_1, \ldots, x_m)$ and ranges $[\underline{x_i}, \overline{x_i}]$ for the unknowns x_i, interval analysis allows to determine simply an interval $[a, b]$, called an *interval evaluation* of f, such that for all x_i in $[\underline{x_i}, \overline{x_i}]$ we have $a \leq f(x_1, \ldots, x_m) \leq b$ i.e. a, b are lower and upper bound of the minimum and maximum of f over the ranges. Usually a, b overestimate the minimum and maximum but the overestimation decreases with the width $\overline{x_i} - \underline{x_i}$ of the range. A major interest of interval analysis is its robustness with respect to round-off errors: even if such errors occurred, the range $[a, b]$ is still guaranteed to include the real minimum and maximum of f.

Such method may be used to determine ranges for the pose parameters being given a time interval $[t_0, t_1]$ and the set $\mathbf{F}$. In turn there ranges will be used to determine a range $[a_i, b_i]$ for the value of each kinematic constraint $\mathcal{C}_i$. If $a_i > 0$, then the constraint is violated at any time in $[t_0, t_1]$, while if $b_i < 0$, then the constraint is satisfied all over the time range. It may occur that due to the overestimation of interval analysis we get $a_i < 0, b_i > 0$, so that we cannot state if the constraint is violated or satisfied. In that case we will bisect the time range in two ranges $[t_0, (t_0+t_1)/2], [(t_0+t_1)/2, t_1]$ and start again the process for each of these ranges, until we determine either that for some time range a constraint is violated or all constraints are satisfied for any range obtained through this bisection process. It may also occur that for a given time interval analysis failed to determine if one (or more) kinematics constraint is satisfied or violated because round-off errors do not allow to determine the sign of the constraints.

It must be noted that we have assumed an analytical form for $\mathcal{C}$ but what is strictly necessary is a method to calculate an interval evaluation of an index that allows one to determine if the constraint is satisfied or violated. For example for closed-chain robot a singularity occurs if the Jacobian matrix is singular. To detect such occurrence we use the sign of the determinant without having its analytical form: only analytical forms of the components of the Jacobian are required [14].

A first extension of this algorithm was to consider uncertainties in the geometric modeling of the robot that is used to formulate the kinematic constraints. In mechanical engineering they correspond to manufacturing tolerances, that are bounded. We use a worst case scenario by assuming that the real value of the geometrical parameters may be any value within known ranges. If there width are relatively small their ranges are used as it for the interval evaluation of the kinematic constraints. In the worst case it may happen that for a given fixed time that the interval evaluation of one (or more) kinematic constraint has a lower negative bound and positive upper bound, thereby not allowing to determine if the constraint is satisfied: in that case the parameters uncertainties are added as new variables and submitted to the bisection process. Note that the same process may be used if we assume *control errors*, i.e. the robot will not follow exactly the planned trajectory, the differences between the trajectory and the robot pose being still bounded.

At this point we are thus able to design an algorithm $\mathcal{A}_P(\mathbf{W_j}, \mathbf{W_{j+1}})$ that returns 1 if the trajectory between the way points satisfies the kinematic constraints, -1 if it violate them or 0 if the trajectory is unsafe (i.e. at some time we cannot determine if all kinematic constraints are

satisfied).

But we may extend this algorithm to deal with set of way points defined by a *box* in the operational workspace. This new algorithm, $\mathcal{A}_{\mathcal{P}}(\mathcal{W}_j, \mathcal{W}_{j+1})$, takes as input two boxes of the operational workspace that define possible poses for the way point $\mathbf{W_j}, \mathbf{W_{j+1}}$ and returns 1 if the trajectory between any poses in the boxes $\mathcal{W}_j, \mathcal{W}_{j+1}$ satisfies the constraint, -1 if all trajectories violate them. Furthermore the algorithm returns 0 if it is not able to complete its task after a limited number of iteration.

B. The optimality criteria

We will assume that we are able to calculate an interval evaluation of the eventual optimality criteria $\mathcal{H}$ (without loss of generality we will assume that the optimality criteria should be positive and minimal). Our purpose will then be to determine a trajectory with a criteria that is at most ϵ away from the minimal value of $\mathcal{H}$, ϵ being a value defined by the end-user.

Now assume that we have determined a trajectory for which the value of $\mathcal{H}$ is l: using the interval evaluation of the criteria we are able to design an algorithm $\mathcal{O}(\mathcal{W}_{1,\dots,m})$ that takes as input a set of boxes for the m way points and returns 1 if the lower bound of the interval evaluation of $\mathcal{H}$ is lower than $l-\epsilon$ (i.e. there may be trajectories having a value of $\mathcal{H}$ that is lower than $l-\epsilon$ with way points in the boxes), -1 if the lower bound of the interval evaluation of $\mathcal{H}$ is greater than $l-\epsilon$ (i.e. even if there are valid trajectories with way points in the boxes, they will have a higher value of $\mathcal{H}$ than the current trajectory). Otherwise the algorithm will return 0.

III. The local planner

The first step of the local planner is to determine if there is valid trajectory with one way point between S, G. The eventual way point will be located in the workspace, hence we have bounds for its coordinates. A *way point box* is a box of the operational workspace that may contain the way point $\mathbf{W_1}$. The workspace will be bisected into way point boxes. For each boxes we will test if the trajectory with as way point the center of the box is valid by using algorithm $\mathcal{A}_P$ on the trajectory components $S\mathbf{W_1}$, $\mathbf{W_1}G$. If not we will try to determine if the current way point box does not contain any valid trajectory by using the algorithm $\mathcal{A}_{\mathcal{P}}$. If this is the case we discard the box, otherwise we bisect it.

If an optimality criteria has been defined and a valid trajectory has been found with a value of $\mathcal{H}$ equal to l, $\mathcal{O}$ will be used to determine if the current way point box may include trajectories with a value of $\mathcal{H}$ lower than $l-\epsilon$. If this is not the case the box will be discarded.

A. Incremental addition of way points

It may occur that there is no valid trajectory with only one way point or that the purpose of the planner is to determine a trajectory whose value of $\mathcal{H}$ is almost optimal, i.e. if a trajectory with m way points with a value of $\mathcal{H}$ equal to l_o has been determined, then even by adding an arbitrary large number of way points we will not find a trajectory with a value of $\mathcal{H}$ lower than $l_o-\epsilon$. For that purpose we will incrementally add way points until either a trajectory (or a better value of $\mathcal{H}$) is found or we are able to determine that the current trajectory is optimal. The size of the way point boxes will increase with the number of way points: when looking for one way point the size of the box is n, $2n$ when looking for two way points and so on.

B. Bisection

Before bisecting it is necessary to choose which variable of the way point box will be bisected. A classical approach for this choice is to select the variable whose range has the largest width. However if the variables have different physical meaning (e.g. coordinates in a reference frame versus rotation angles) it may be appropriate to affect a weight to each variable. If an optimality criteria has been defined another possibility is to bisect each variable and to compute the interval evaluation of $\mathcal{H}$ for each of the resulting boxes. Then the variable which has the largest influence on the width of the interval evaluation of $\mathcal{H}$ will be retained as the bisected variable.

C. The algorithm

We present here the algorithm when an optimality criteria has been defined. The number of way points will be denoted N and the algorithm starts with $N=1$. A flag T, initially set to 0, will be set to 1 if a valid trajectory has been found. The value of $\mathcal{H}$ for this trajectory will be denoted $\mathcal{H}_T$. The algorithm will process a list $\mathcal{L}$ of way point boxes $\mathcal{B} = \{\mathcal{W}_1, \dots \mathcal{W}_N\}$. The $i-th$ element of the list will be $\mathcal{B}_i$ and r will denote the total number of way point boxes in the list. The mid point of a way point box is obtained by taking the mid point of all the ranges of the variables in the box: it correspond to a set of fixed poses and will be written as $Mid(\mathcal{B}_i)$. The value of the optimality criteria obtained for a trajectory with N way point will be denoted l_o^N (l_o^0 will be equal to $+\infty$). When starting the algorithm we have $T=0$ and there is one way point box $\mathcal{B}_1$ in $\mathcal{L}$, whose ranges correspond to the ranges of the workspace $\mathcal{E}$. The Cartesian product of N box $\mathcal{E}$ will be denoted $\mathcal{E}^N$.

while true **do**
  $r = i = 1$, $\mathcal{L} = \{\mathcal{E}^N\}$
  if $T=1$, then $l_o^N = l_o^{N-1}$
  while $i \leq r$ **do**
    if $T=1$ and $\mathcal{O}(\mathcal{B}_i) = -1$, then $i = i+1$, **next**
    if $\mathcal{A}_{\mathcal{P}}(\mathcal{B}_i) = -1$, then $i = i+1$, **next**
    if $\mathcal{A}_P(Mid(\mathcal{B}_i)) = 1$, then
      $l_w = \mathcal{H}(Mid(\mathcal{B}_i))$
      if $T=1$, $l_w < l_o^N$, then $l_o^N = l_w$
      if $T=0$, then $T=1$, $l_o^N = l_w$
    bisect $\mathcal{B}_i$, $r = r+2$, $i = i+1$, **next**
  end-do
  if $T=1$, $l_o^N < l_o^{N-1} - \epsilon$, then $N = N+1$, **next**
  if $T=1$, $l_o^N = l_o^{N-1}$, then **break**
  if $T=0$, then $N = N+1$, **next**
end-do

As for any interval-based algorithm the efficiency of the algorithm will be improved by adding *filtering* methods, i.e. methods that reduces the size of a way point box without using bisection. We use standard interval analysis filtering methods (see for example [7]) but also adapted methods as will be shown in the examples.

Note that exiting the algorithm as soon as adding one way point does not allow to improve significantly l_o (i.e. leads to a decrease larger than ϵ) does not exactly allow to certify that a close approximation of the best trajectory has been found. For example we way imagine cases where adding new way points will improve $\mathcal{H}$ by $\epsilon/2$ and hence adding 3 way points will lead to a better trajectory. In practice however this will imply that the global planner has performed poorly.

A first drawback of this algorithm is that its complexity quickly increase with the number of way points. This will be confirmed by the example: determining a trajectory with one or two way points is fast but the computation time drastically increase for 3 way points. But we may reduce the computation time for 2 (or more) way points with an incremental use of the local planner, an approach that we will call the *intermediary step method*. This will be illustrated with an example in which a trajectory with one way point has been found. Let $S, \mathbf{W_1}, G$ be the current trajectory. We use the local planner with a threshold equal to, for example $\epsilon/2$ to determine if there is a trajectory with one way point $\mathbf{W_1}'$ between $S, \mathbf{W_1}$ that reduces the value of $\mathcal{H}$ compared to its value in the current trajectory. If this is the case we get the trajectory with 2 way points $S, \mathbf{W_1}', \mathbf{W_1}, G$ that has already a better value for $\mathcal{H}$ than the one way point trajectory. The process may be repeated with $\mathbf{W_1}, G$ and we may select the two way points trajectory that leads to the best improvement of $\mathcal{H}$. We get therefore a better initial trajectory for the local planer when looking for a trajectory with 2 way points.

A second drawback of this algorithm is that it does not have a memory. Indeed when checking a trajectory with two or more way points we will examine possible location of $\mathbf{W_1}, \mathbf{W_2}$ that have already been determined as non valid when we have looked for a trajectory with one way point.

IV. Examples

Four our tests we have used a 6 d.o.f. parallel robot that is presented in the next section.

A. Parallel robot

As example of complex 6-dof closed-loop robot we will consider a Gough platform (figure 2). The fixed frame $(O, \mathbf{x}, \mathbf{y}, \mathbf{z})$ will be called the *base frame* while a mobile frame $(C, \mathbf{x_r}, \mathbf{y_r}, \mathbf{z_r})$ attached to the platform will be called the *mobile frame*. The pose of the platform will be parameterized by the location of C in the base frame and 3 angles will be used to define the orientation of the mobile frame with respect to the base frame.

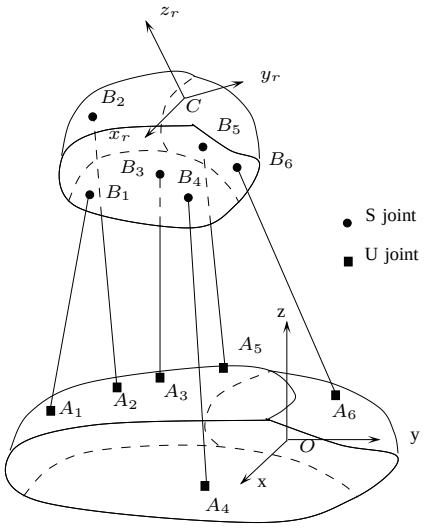

Fig. 2. A Gough-Stewart platform

Motion of the platform are obtained by changing the length of the 6 legs, that are attached on the base at A_i and on the platform at B_i. The coordinates of the vector $\mathbf{OA_i}$ in the reference frame and the coordinates of $\mathbf{CB_i}_r$ in the mobile frame are known. The legs are restricted to have a minimal and maximal length, thereby restricting the workspace of this robot. Being given the location of the center C of the platform in the reference and the rotation matrix $\mathbf{R}$ between the mobile and reference frame, the length ρ of a leg is determined as

$$\rho^2 = ||\mathbf{AO} + \mathbf{OC} + \mathbf{RCB}_r||^2 \tag{1}$$

and we should have

$$\rho_{min}^2 \leq \rho^2 \leq \rho_{max}^2$$

The shape of the workspace due to this limitation is quite complicated [12] and planning a trajectory to fully lie in this workspace is a complex task. Other kinematic constraints may be considered as well: limitation of the passive joints motion at A, B [16], singularity avoidance [1], [5], [6], [8], [15] or leg interference [3], [13]. We will consider here first only the limitation on the leg lengths, limits on the motion of the passive joints.

The optimality criteria that we will use is the length of the trajectory of the center C of the platform. This allows to use a dedicated filtering method that is available as soon as a trajectory of length l with one way point has been determined. Indeed any potential way point that will lead to a smaller trajectory length (i.e. smaller than $l - \epsilon$) will lie within the ellipsoid whose border is defined as the set of points M such that the sum of the distances from M to S, G is equal to $l - \epsilon$. The bounding box E of this ellipsoid may be calculated and then its intersection with the way point box. Such intersection may lead to multiple way point boxes but for simplicity we just compute this intersection if it will lead to a unique box, that will be used as new way point box.

B. Implementation and tests

To test the motion planning algorithm we have used the C++ interval arithmetic package `BIAS/Profil` and some components of our interval analysis library `ALIAS`.

We assume first that the geometry of the robot is perfectly known (i.e. the location of the A_i, B_i are exact as indicated in table I). The minimal and maximal leg lengths

	x_A	y_A	z_A	x_B	y_B	z_B
1	-9	9	0	-3	7	0
2	9	9	0	3	7	0
3	12	-3	0	7	-1	0
4	3	-13	0	4	-6	0
5	-3	-13	0	-4	-6	0
6	-12	-3	0	-7	-1	0

TABLE I

COORDINATES OF THE A, B POINTS FOR THE TEST ROBOT

are 52.249605 and 55.749605.

The various poses are defined by the 3 coordinates of C followed by the three rotation angles. The start point S is (0,0,52.1,0,0,0) and the goal G is (11,5,52.1,0,0,0) as presented in figure 3. Note that point S, G are chosen with the same orientation and in an horizontal plane just for allowing an easy representation of the workspace. The distance between S, G is 12.083. It may been that the workspace has voids in its interior. In these first trials we consider only the limitation on the leg lengths as kinematic constraints. We first try to determine a trajectory that keep the rotation angles to (0,0,0). Note that in this case the global planner should have provided more sampling points. The threshold ϵ for the length of the trajectory is fixed to 0.3. Trajectories with 1, 2 and 3 way points are presented

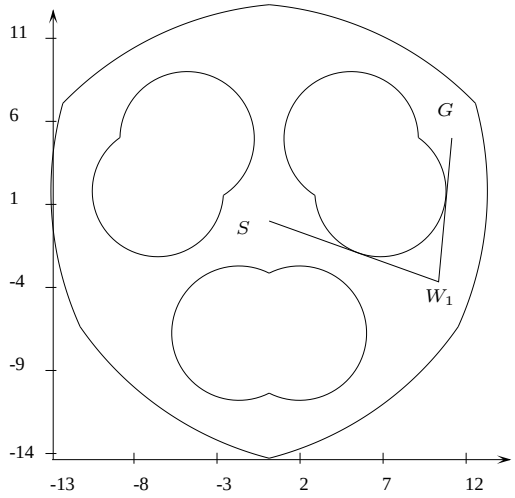

Fig. 3. A cross-section of the workspace with the starting point S, the goal point G and one way point W_1

in figure 4: their lengths are respectively 19.5373, 17.1118, 16.7887 and these trajectories are established respectively in 1 second, 17 seconds and 11 mn 44 seconds on a DELL D400 laptop. As expected the computation time grows quickly with the number of way points. But the computation time does not significantly increase if we add as kinematic constraint that the determinant of the inverse Jacobian matrix has a value larger than a given threshold for any pose on the trajectory in order to ensure that the trajectory lies on a given kinematic branch. If we

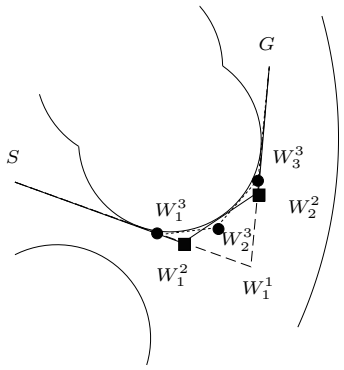

Fig. 4. Trajectories with 1, 2 (squared way points) and 3 (circled way points) way points.

now relax the constraint that the trajectory should lie in the plane $z = 52.1$ by allowing a z value in the range [50,55] while keeping the same orientation we found out in 0.26 second that the trajectory with one way point at (5.562,2.5,52.5351,0,0,0) has a length of 12.1144. As the minimal value of $\mathcal{H}$ is 12.083, the algorithm stops as adding new way point cannot make the length decrease by more than 0.3. Setting the threshold ϵ to 0.01 allows to determine a trajectory with length 12.0917 in a computation time of 1.2 seconds.

We may also keep the trajectory in the same plane while allowing to change the platform orientation. For the same trajectory and allowing the orientation angles to lie in the range [-5,5] degree, we find a one way point trajectory of length 12.3967 in 15 seconds and the planner immediately determine that adding a new way point will not shorten the trajectory length by more than 0.3.

We may also have to deal with uncertainties in the robot modeling. We assume a tolerance error of $\pm$ 0.01 on each coordinates of the A_i, B_i meaning for example that the x coordinate of A_1 may have any value in the range [-9.01,-8.99]. A trajectory with one way point and length 21.2389 is found in 8 seconds, with two way points and length 17.8501 in 5 minutes and 29 seconds. For three way points a trajectory of length 17.4191 is found in 4h27mn49s. In the later case by using the *intermediary step method* we may reduce this time to 3h41mn and the intermediary path allows to get an initial three way point trajectory of length 17.6235 in 16mn10s. These trajectories are presented in figure 5. If we allow z to lie in the range [50,55] a minimal length trajectory is found in 2.65s.

Assume now that we have limits on the passive joint motion. To model this limits for the joints at A_i we assume that the angle between the $i-th$ leg and a fixed direction defined by the unit vector $\mathbf{n_i}$ should be lower than a

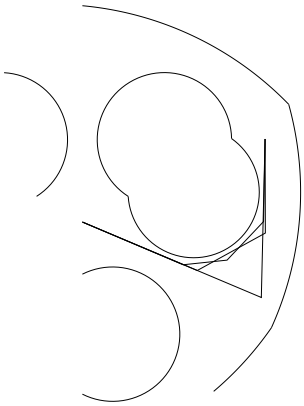

Fig. 5. Trajectories with one to three way points when the coordinates of A_i, B_i have a tolerance of $\pm$ 0.01

threshold μ_i. This constraint may be written as

$$|\frac{\mathbf{A_i B_i . n_i}}{\rho_i}| \leq \cos(\mu_i) \quad (2)$$

We consider the trajectory between S (0,0,52.2) and G (-8,5,52.2) and assume that the angle between the vertical direction and the legs should not exceed 17 degrees. Without considering these constraints we find a trajectory of length 16.3498 with one way point and of length 13.9694 with two way points (figure 6). But if the passive constraints are

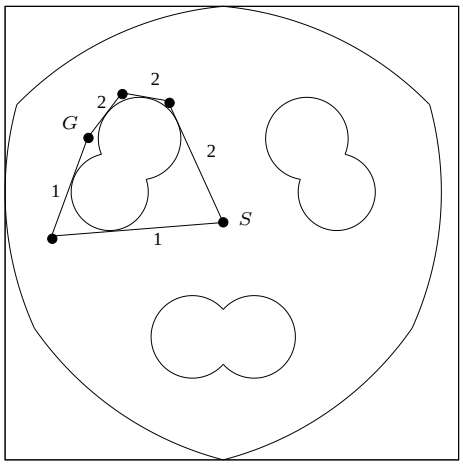

Fig. 6. Trajectories with one (1) and two (2) way points when the constraints on the passive joint limits are not considered.

taken into account the algorithm found out in 3 seconds that there is no trajectory with only one way point. The trajectory with two way points is also not satisfactory as part of it violate the joint limits. But a two way point trajectory that takes into account the joint limits may be found with a length of 14.1181 with a threshold on $\mathcal{H}$ of 0.1.

V. Conclusion

Safe motion planning of closed-loop robot requires to verify complex kinematic constraints apart of the classical obstacle avoidance problem. Global motion planner have the advantage to provide a draft trajectory but a local planner must then be used to provide a certified trajectory (i.e. such that the kinematic constraints are satisfied all along the trajectory). The local planner proposed in this paper is based on interval analysis and allows both to certify the trajectory and to manage possible uncertainties in the robot modeling.

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